Integrable Systems in Pure and Applied Mathematics

Conference in honour of Boris Dubrovin's 60th birthday

ABSTRACTS BOOKLET

Semiclassical Nonlinear Schrödinger and Painlevé; Peregrine breather and poles of the tritronqée solution.

M. Bertola
Concordia University, Montreal, Canada

Abstract

In the semiclassical (zero-dispersion) limit of the one-dimensional focusing Nonlinear Schrödinger equation (NLS) with decaying potentials there is a first time where the genus-zero dispersionless approximation fails due to a gradient catastrophe. I will address the (scaling) behaviour in a full neighbourhood D of this point of gradient catastrophe (x_0, t_0), both in the genus-zero and two regions. This allows us to address and partially verify a conjecture of Dubrovin-Grava-Klein about the role of the integrale tritronquée in the description of the asymptotic behaviour, establishing a universality with respect to a large class of initial datum.

I will also go beyond the prediction of the conjecture, establishing the further interestingly simple universal behaviours;

(1) the maximum amplitude of each spike in the genus two region (near the gradient catastrophe) is exactly three times the one at the gradient catastrophe;
(2) the shape of each spike is universally the one of the rational breather solution (aka rogue wave) to the NLS (due to Peregrine);
(3) the spikes are in one-to-one correspondence with the poles of the *tritronquée* solution of the Painlevé I (P1).

The method is based upon the nonlinear steepest descent method and allows us to conjecture that the P1 hierarchy occurs at higher degenerate catastrophe points and that the amplitudes of the spikes are odd multiples of the amplitude at the corresponding catastrophe point.

I will try to describe the phenomenon qualitatively in the first half and possibly show some key ingredient of the proof in the second half; a technical but quite crucial and -to my knowledge- new ingredient is the fact that the local parametrix (constructed from the ψ function of Painlevé I) needs to be studied in a neighborhood of the pole of the Painlevé transcendent.

The whole talk is based upon joint work with Alexander Tovbis.

Discrete conformal mappings and ideal hyperbolic polyhedra

A. I. Bobenko
TU Berlin, Germany

Abstract

Two triangulated surfaces are considered discretely conformally equivalent if the edge lengths are related by scale factors associated with the vertices. This simple definition leads to a surprisingly rich theory featuring Moebius invariance, the definition of discrete

conformal maps as circumcircle preserving piecewise projective maps, and two variational principles. We show how literally the same theory can be reinterpreted to addresses the problem of constructing an ideal hyperbolic polyhedron with prescribed intrinsic metric. This synthesis enables us to derive a companion discrete uniformization theory. (joint with U. Pinkall and B. Springborn)

Stringy enumerative geometry and integrable hierarchies

A. Brini
Université de Genève, Switzerland

Abstract

A long-standing issue in Gromov-Witten theory is its conjectural relationship with infinite dimensional integrable systems. This connection has had a lot of circumstantial evidence in its favour, yet explicit constructions seem very hard to obtain. In this talk I will present a number of results concerning the integrable hierarchies that govern à la Witten-Konstevich the Gromov-Witten theory of some toric Calabi-Yau threefolds; among their consequences, the construction of a genuinely A-model-based formalism for the computation of open string potentials of toric CY3 will be outlined.

The Riemann-Hilbert approach to the transition from the Pearcey to the Airy process

M. Cafasso
Concordia University, Montreal, Canada

Abstract

I will consider the gap probability for the Airy and the Pearcey processes. Using a Riemann-Hilbert approach I will show that the Pearcey process, in a large gap/large time limit, factorizes in two independent Airy processes. This talk is based on a joint work with Marco Bertola.

TBA

G. Carlet
Universidade de Coimbra, Portugal

Abstract

Critical asymptotics for the KdV equation in the small dispersion limit

T. Claeys
Université de Lille 1, France

Abstract

I will discuss the small dispersion limit for the KdV equation in critical regimes which lie between the regions of regular and oscillatory behavior. Three different critical cases occur, and two of them lead to asymptotic expansions in terms of Painlevé transcendents. I will also give an overview of the Riemann-Hilbert approach used to obtain the critical asymptotic expansions. The talk will be based on joint work with Tamara Grava.

Classical geometry and integrability – billiards and tops

V. Dragovic
MI SANU, Belgrade, Serbia/GFM University of Lisbon, Portugal

Abstract

We review classical and modern results on geometry of pencils of quadrics and related integrable billiard dynamics. We present a new view on the celebrated Kowalevski top, based on the Darboux coordinates, n-valued Buchstaber-Novikov groups and a new notion of discriminantly separable polynomials. We discuss possible integrable generalizations of systems of Kowalevski type.

On the Tetrahedrally Symmetric Monopole

V.Enolski
Heriott Watt, UK

Abstract

We study $SU(2)$ BPS monopoles with spectral curve of the form $\eta^3+\chi(\zeta^6+b\zeta^3-1)=0$. Previuos work has established a countable family of solutions to Hitchin's constraint that L^2 is trivial on such a curve. Here we establish that the only curve of this family that yields BPS monopole correspond to tetrahedrally symmetric monopoles. We introduce several new techniques making use of a factorization theorem of Fay and Accola for theta functions, together with properties of the Humbert variety. The geometry leads us to a formulation purely in terms of elliptic functions.

Poisson Pencils and (Algebraic) Integrability

G. Falqui
Università degli studi di Milano-Bicocca, Italy

Abstract

I plan to review and some properties of the bi-(or multi) Hamiltonian geometry of integrable (finite dimensional) systems of Toda and - possibly - Gaudin type. In particular, I shall show how some methods of the geometry of specific classes of bi-Hamiltonian

manifolds can be successfully used in the algebro-geometrical approach to these integrable systems. A couple of significant examples will be discussed as applications of such techniques.

Integrable Hamiltonian systems of hydrodynamic type in 2+1 dimensions

E. Ferapontov
Loughborough University, UK

Abstract

A complete description of integrable Hamiltonians corresponding to the existing types of two-component Poisson brackets in 2+1 dimensions will be discussed. The talk is based on joint work with A. Moro, V Sokolov and N. Stoilov.

Numerical Study of the Kadomtsev-Petviashvili Equation

C. Klein
Institut de Mathématiques de Bourgogne, France

Abstract

We present an accurate numerical study of the Kadomtsev-Petviashvili (KP) equation. In particular, we are concerned with the small dispersion limit of this model, where no comprehensive analytical description exists so far. To this end, we first study a similar highly oscillatory regime for asymptotically small solutions, which can be described via the Davey-Stewartson system. In a second step, we investigate numerically the small dispersion limit of the KP model in the case of large amplitudes. Similarities and differences to the much better studied Korteweg-de Vries situation are discussed as well as the dependence of the limit on the additional transverse coordinate. We also study the stability of exact solutions to the KP equation, and the appearence of blowup in generalized KP equations.

Singular sectors, degenerate critical points and Euler-Poisson-Darboux equations for the dcKdV hierarchy, dToda hierarchy and Hermitian matrix model

B. Konopelchenko
Università degli Studi di Lecce, Italy

Abstract

Structure of the spaces of hodograph solutions for the 1-layer Benney hierarchy, dispersionless Toda hierarchy and large N limit of the Hermitian random matrix model is discussed. It is shown that the hodograph equations for these systems describe the critical points of functions W which obey the Euler-Poisson-Darboux equation $E(a, a)$

with $a = 1/2$ or $a = -1/2$. Singular sectors for these hierarchies are associated with the degenerate critical points of functions W. The fact that functions W obey the Euler-Poisson-Darboux equation allows us to characterize the singular sectors of these models completely and to find the critical behaviour of the asymptotic eigenvalue density near the endpoints for the Hermitian matrix model in zero genus case. (joint work with L. Martinez Alonso and E. Medina (Spain))

Isomonodromic tau function on the space of admissible covers

D. Korotkin
Concordia University, Montreal, Canada

Abstract

The isomonodromic tau function of the Fuchsian differential equations associated to Frobenius structures on Hurwitz spaces can be viewed as a section of a line bundle on the space of admissible covers. We study the asymptotic behavior of the tau function near the boundary of this space and compute its divisor. This yields an explicit formula for the pullback of the Hodge class to the space of admissible covers in terms of the classes of compactification divisors. (joint work with Alexey Kokotov and Peter Zograf)

TBA

I. Krichever
Columbia University, New York, USA

Abstract

The Kowalevski Top 121 years later.

F. Magri
Università degli studi di Milano-Bicocca, Italy

Abstract

I will provide some novel insights into the paper of Sophie Kowalevski of 1889. In particular I will discuss the geometric structures which are behind the separability of the Kowalevski top.

Painlevé I and the Cubic Oscillator

D. Masoero
SISSA, Trieste

Abstract

Poles of solutions to the Painlevé-I equation are intimately related to the theory of the cubic anharmonic oscillator. In particular, poles of integrale tritronquee are in 1-1 correspondence with cubic oscillators that admit the simultaneous solutions of two quantization conditions. We analyze this pair of quantization conditions by means of a suitable version of the complex WKB method. Eventually We introduce a new tool, the Deformed TBA (Deformed Thermodynamic Bethe Ansatz), to analyze the monodromy problem of the cubic oscillator.

Darboux transformation method for the integrable PDEs: from solitons to rogue waves

V. Matveev
Institut de Mathématiques de Bourgogne, France

Abstract

We describe some basic features of the Darboux transformation method introduced by the speaker more than 30 years ago allowing a purely algebraic construction of the large classes of explicit solutions to the numerous integrable linear and nonlinear PDEs and their difference versions. We illustrate a power of the method by the most important examples including constructions of various important singular and nonsingular solutions to the KdV , KP-I and KP-II equations. We also briefly discuss some special motions of the Calogero-Moser particles described by means of the Schur polynomials, the multi-rogue waves solutions to the focusing NLS equation, connected to the special class of the real nonsigular rational solutions to the KP-I equation, and the multi positons solutions to the KdV equation.
(The speaker was supported by ANR via the program ANR grant HOSDINA: ANR-09-BLAN-0117-01.)

Isomonodromic deformations and twisted Yangians arising in Teichmüller theory

M. Mazzocco
Loughborough University, UK

Abstract

In this talk we build a link between the Teichmüller theory ofhyperbolic Riemann surfaces and isomonodromic deformations of linear systems whose monodromy group is the Fuchsian group associated to the given hyperbolic Riemann surface by the Poincaré uniformization. In the case of a one–sheeted hyperboloid with n orbifold points we show that the Poisson algebra $\mathfrak{D}_n$ of geodesic length functions is the semiclassical limit of the twisted q–Yangian $Y'_q(\mathfrak{o}_n)$ for the orthogonal Lie algebra $\mathfrak{o}_n$ defined by Molev, Ragoucy and Sorba. We give a representation of the braid group action on $\mathfrak{D}_n$ in terms of an adjoint matrix action. If we have time, we shall characterize two types of finite–dimensional Poissonian reductions and give an explicit expression for the generating

function of their central elements. Finally, we interpret the algebra $\mathfrak{D}_n$ as the Poisson algebra of monodromy data of a Frobenius manifold in the vicinity of a non-semisimple point.

Asymptotic analysis of Riemann-Hilbert problems using d-bar methods, and an application

K. McLaughlin
University of Arizona Abstract

I'll explain how to use some basic tools from harmonic analysis to simplify some aspects of the asymptotic analysis of Riemann-Hilbert problems. If there is time, I will explain an application: how long-time asymptotic calculations for the NLS equation can be used to study delta-function initial data. (joint work with Jason Newport)

On Dyson's non-intersecting Brownian motions

P. van Moerbeke
Université de Louvain, Louvain-la-Neuve, Belgium

Abstract

Dyson introduced dynamics in random matrix models, by letting the entries of GUE evolve according to Ornstein-Uhlenbeck processes. He moreover shows that the spectrum of this random matrix evolves according to non-intersecting Brownian motions.

Do the interlaced spectra of two or more consecutive principal minors lead to a diffusion? I show that this is so for two consecutive minors, but not for more. I shall also address limiting questions when the number of particles becomes very large, leading to new universality classes and mention a number of open problems.

Differential operators, generated by Young diagrams

S. Natanzon
Indipendent University of Moscow, Russia

Abstract

We correspond some differential operator to any Young diagram. The operators have the simplest form if expressed in terms of the matrix Miva-variables. Shur functions form full system eigenfunctions for our operators. The operators generate commutative algebra, that is isomorphic to algebra of skew symmetric functions. The operator is Cut-and-Join operator from theory of Hurwitz numbers. (joint work with A.Mironov and A.Morozov)

TBA

S. P. Novikov
University of Maryland, USA

Abstract

Equivariant GW theory of local P1, mirror model and Ablowitz-Ladik hierarchy

P. Rossi
Université Pierre et Marie Curie - Paris 6, France

Abstract

In a joint work in progress with A.Brini and G.Carlet we study the mirror Landau-Ginzburg model for the equivariant Gromov-Witten theory of local P1 and clarify its relation with Ablowitz-Ladik hierarchy of PDEs (some generalization of Dubrovin's duality of Frobenius manifolds). We then use it to compute explicitly the topological J-function via oscillating integrals and relate it to the dispersionless hamiltonians of the AL hierarchy from the Lax formalism.

KdV like equations, Poisson Lie groups and differential Galois theory

M. Semenov-Tian-Shansky
Institut de Mathématiques de Bourgogne, France

Abstract

The projective geometry approach to Schröedinger equations on the circle and differential Galois theory are combined with the theory of Poisson Lie groups to construct a natural Poisson structure on the space of wave functions (at the zero energy level). Applications to KdV-like nonlinear equations are discussed. The same approach applied to difference operators on a lattice yields an extension of the lattice Poisson Virasoro algebra.

Automorphisms of cluster algebras

V. Shramchenko
Sherbrooke University, Canada

Abstract

Cluster algebras were introduced in 2002 by S. Fomin and A. Zelevinsky. These algebras are combinatorial structures which appear in different contexts, from the theory of Lie groups to Teichmüller theory. Ideal triangulations of boarded surfaces with punctures give rise to some cluster algebras. We introduce and study the notion of a cluster automorphism. In the case of cluster algebras arising from triangulations of surfaces, we relate the group of cluster automorphisms to the mapping class group of the corresponding surface. (joint work with I. Assem and R. Schiffler)
I will start with a short introduction to the theory of cluster algebras.

On two-point correlation function in quantum sine-Gordon model

F. Smirnov
Laboratoire de Physique Théorique et des Hautes Energies, UPMC Paris, France

Abstract

At the free fermion point the two-point function of exponential operators for the sG model is given by Painlevé III transcendent. The important consequence of this is that the asymptotics at long and short distances is under complete control. I shall explain that this is the case for the interacting model. The long distance asymptotics is completely fixed by the form factors expansion. I computed the form factors 25 years ago. The short distance asymptotics is controlled by CFT perturbation theory and the one-point functions, which we computed recently with M. Jimbo and T. Miwa.

Eventual identities and almost duality at $d = 1$

I. Strachan
Glasgow University, UK

Abstract

In an article dedicated to Novikov on the occasion of his 65th birthday, Dubrovin introduced the idea of almost duality. By twisting the normal Frobenius multiplication by the inverse of the Euler vector field one obtains structures which satisfy - almost - the axioms of a Frobenius manifold.

Manin generalized this idea by using what he called eventual identities, but the characterization of these vector fields was left open. In the first part of the talk the characterization of such eventual identities is given. In the second, properties of almost dual Frobenius manifolds at $d = 1$ are discussed: here certain invariant properties under the modular group replace the usual scaling symmetries. (joint work with L.David and E. Morrison)

On geometry of poles of monodromy-free potentials

A. Veselov
Loughborough University, UK

Abstract

A meromorphic potential $u(z)$ is called monodromy-free if the corresponding Schröedinger equation $-y'' + u(z)y = Ey$ has all solutions meromorphic in the whole complex plane for all values of E. An important class of such $u(z)$ is given by the finite-gap potentials. I will discuss geometry of the pole configurations of the monodromy-free potentials in some special cases, partly following recent joint work with Felder and Hemery.

On symmetries of the WDVV equations

Y. Zhang
Tsinghua University, Beijing

Abstract

For two solutions of the WDVV equations that are related by a discrete symmetry of the equations given by Dubrovin, we show that the associated Principal Hierarchies of integrable systems are related by certain reciprocal transformation, and the tau functions of the Principal Hierarchies are either identical or related by a Legendre transformation. We also consider relationships between the Virasoro constraints and topological deformations of the Principal Hierarchies.

Square-tiled cyclic covers and Lyapunov exponents of the Hodge bundle

A. Zorich
Université de Rennes, France

Abstract

We start with a discussion of square-tiled surfaces and its Teichmüller curves. As a more specific result we show that for any arithmetic Teichmüller curve corresponding to a cyclic cover over a projective line branched at four points one can explicitly compute the spectrum of Lyapunov exponents of the Hodge bundle along the Teichmüller flow.